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AN ESTIMATION OF KOON SYSTEMS AVAILABILITY USING A SIMULATION METHOD

Key words: availability, koon systems availability, simulation models, Markov processes.

Abstract: The paper presents a developed simulation model for an availability estimation of renewable technical systems characterised by koon reliability structures. The proposed solution allows one to describe the availability of systems consisting of elements whose time-failure and renewal time probability distributions are, e.g., exponential, Weibull, or normal. This is the main advantage of the model. The results obtained from the use of the model, when exponential probability distribution applied, were verified by their comparison with the results of using typical, widely-known tools for a reliability and/or availability estimation of renewable systems (Markov processes). Additionally, the exemplary results of the simulations conducted for non-exponential probability distributions of time-failure and renewal time were presented. The useful feature of the developed model is the ability to estimate the availability of real systems and to verify whether they provide its required level. Such requirements are applied in many industries and services. Another important feature of the model is the possibility of searching for the most efficient method of increasing the availability of the system to its required level.

Prognozowanie gotowości układów typu kzn metodą symulacyjną

Słowa kluczowe: niezawodność, gotowość układów typu kzn, modele symulacyjne, procesy Markowa.

Streszczenie: W artykule przedstawiono opracowany model symulacyjny przeznaczony do oceny gotowości odnawialnych układów technicznych charakteryzujących się strukturami niezawodnościowymi typu kzn. Proponowane rozwiązanie pozwala charakteryzować gotowość układów składających się z elementów, których czasy pracy do uszkodzenia i czasy odnowy opisywane są różnymi rozkładami prawdopodobieństwa, np.: wykładniczym, Weibulla lub normalnym. Stanowi to główną zaletę prezentowanego modelu. Wyniki obliczeń uzyskane przy zastosowaniu opracowanego modelu, gdy wykorzystywany był rozkład wykładniczy, zweryfikowano poprzez ich porównanie z wynikami obliczeń za pomocą typowych, powszechnie znanych narzędzi do oceny niezawodności lub gotowości systemów odnawialnych (procesów Markowa). Dodatkowo, w artykule przedstawiono przykładowe wyniki symulacji przeprowadzonych dla rozkładów prawdopodobieństwa czasu pracy i czasu odnowy innych niż wykładniczy. Użyteczną cechą opracowanego modelu jest możliwość szacowania gotowości rzeczywistych systemów i sprawdzania, czy zapewniają one jej wymagany poziom. Tego rodzaju wymagania występują w wielu gałęziach przemysłu i obszarach usług. Kolejną ważną cechą modelu jest możliwość poszukiwania najbardziej efektywnej metody zwiększania gotowości systemu do wymaganego poziomu.

Introduction

The complex technical systems which are in use nowadays must provide the ability to perform their functions at any time, and often in varying ways according to the current requirements and needs. One of the ways to meet the requirements is the use of redundant systems that are often designed as the koon (*k* out of *n*) systems. The specific structure of the koon system used in a given case depends on the requirements relating to the reliable functioning of the entire system and the technical and economic capacity to maintain additional redundant objects (cf. [7]). In order to measure the performance of the system, the availability coefficient may be applied. It characterises the probability that, at the time *t*, a technical system remains in its operation state making its proper functioning is possible. Examples of such systems are pump systems in water and sewage pumping stations, systems of fans widely used in industry and fire protection, power block systems in CHP plants, railway signalling systems, as well as vehicle fleet systems in goods transport and public transport companies [2, 3, 5].

Availability (*A(t)*), alongside reliability and durability, is one of the basic characteristics of a technical object that is relevant to its user. For a renewable object, availability is defined as the probability that at any time *t* the object will be in an operational state; therefore, it will be able to perform the functions to which it is intended. Depending on object's time-failure probability distribution, the value of non-stationary availability coefficient (*A(t)*) has an initial oscillation time, but, for a sufficiently long time period, it tends to a constant value [1, 9]. This constant value is described as the object's stationary availability coefficient (*A*), presented with the use of Formula 1 [1]:

$$A = \lim_{t\to\infty} A(t) = \frac{E(T)}{E(T)+E(\Theta)} \tag{1}$$

where

E(T) – expected value of time when the object remains in its operation state

E(Θ) – expected value of time when the object remains in its failure state.

When the object's operation and renewal times are described using exponential distribution, the stationary availability coefficient may be presented using Formula 2 [1]:

$$A = \lim_{t\to\infty} \frac{\mu + \lambda exp\left[-(\mu+\lambda)t\right]}{\mu+\lambda} = \frac{\mu}{\mu+\lambda} \tag{2}$$

where

μ – repair rate of the object,

λ – failure rate of the object.

Safety Related Systems form a wide range of technical systems where availability is especially important. The availability of these systems determines the safety integrity levels (SIL) which may be provided according to their operation. However, due to the specifics of their functioning, the estimation of availability (or non-availability) of such systems is done using specially developed methods [8, 10]. These take into consideration the objects' diagnostic processes conducted during their operation, the classification of their potential failures, which are detectable or undetectable when performing diagnostic tests as well as the possibility of the occurrence of so-called common cause failures. What matters most is the fact that these methods may be applied only in the circumstances when physical objects are in a normal operation period and their failure rate and renewal intensity are constant over the time.

For the complex technical systems, consisting of renewable objects and incorporating redundant elements, like koon systems, the estimation of their availability index is difficult. In cases when failure and repair rates of the objects are constant in time, Markov processes may be successfully applied to estimate the availability of such systems [8, 10]. In other situations, estimation is possible using digital simulation methods [6, 9].

Providing the required system's reliability or availability is possible, not only by developing its structure to be redundant, but also by the application of its preventive maintenance. Possible effects which could be obtained in this case when estimating the reliability of the systems incorporating heterogeneous objects characterised by different statistical distributions of probability of their operation time till failure were presented in [4].

The paper presents a method for any koon systems availability coefficient calculation when the elements of the system are renewable. The estimation of the availability coefficient values is done using computer simulation. Its accuracy was verified for the selected simple structures of koon systems, which could be analysed analytically applying Markov processes. The developed method allows one to rapidly estimate the availability of the koon systems incorporating homogeneous and heterogeneous objects as well as to search for the koon system's structure that provides the required availability level of the analysed system.

1. Developed simulation model assumptions

Knowing the cumulative distributions of system's elements times till failures in operation and reserve states, and the cumulative distribution of elements' renewal time in renewal state, the functioning of the system may be simulated. The allowed transitions between operation, renewal, and reserve states of elements as well as the possible transitions between operation and failure states of the entire system, as assumed in the presented model, are shown in Fig. 1.

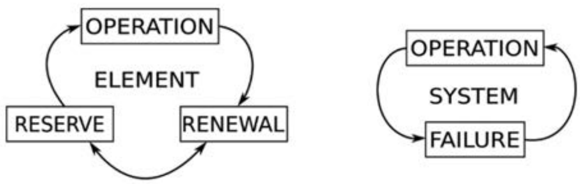

Fig. 1. Transitions of states of the system and its elements

The transition between element states occurs if the probability set by Formula 3 is greater than the random number with a uniform distribution. The transitions between the system states are determined by the changes of the states of its elements.

$$P = \frac{F\left(t_i + \Delta t_i\right) - F\left(t_i\right)}{1 - F\left(t_i\right)} \tag{3}$$

The simulation is conducted as an iterative process. Each iteration is a hypothetical course of the functioning of the analysed system. An exemplary diagram of a single iteration is shown in Fig. 2.

The meanings of the symbols used in the Fig. 2 are as follows:

O – system in its operation state
F – system in its failure state
k – number of elements inevitable for system's operation
<k – less than *k* elements in operation
>n-k – more than *n-k* elements in renewal
n – number of all of the elements of the system
m – number of the currently available redundant elements ($m \le n-k$)
$E_{OPERATION}$, $E_{RESERVE}$, $E_{RENEWAL}$ – elements in operation, reserve, and renewal states.

It is assumed that, at the beginning of an iteration, *k* elements operate so the entire system is in operation state and *n-k* elements are in reserve state. An additional advantage of the developed model is the provided possibility of conducting simulation experiments taking into account the differences in age of the individual elements of the system at the beginning of an experiment.

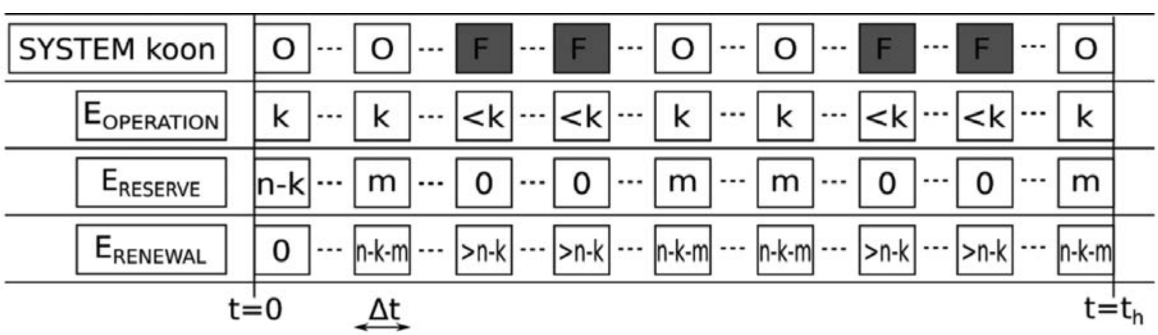

Fig. 2. An exemplary diagram presenting system functioning during an iteration

An iteration is divided into steps. In each step, the age of the elements and system is increased by the value of the time step Δt. In case of the element's state change, the age of the element is set as 0, which represents, among other things, the replacement of the element with a new one. An iteration continues until the age of the system is greater or equal to the set time horizon t_h.

The lengths of the time intervals when the system is in an operation state are recorded. After conducting the simulation process and all the iterations (in practice, at least 10000), one may determine the availability of the system for each moment dividing the number of iterations when the system was at that time in an operation state by the number of all iterations *i*.

2. Application of Markov processes for availability estimation of koon systems

When applying Markov processes for availability evaluation, the possible reliability states of the analysed system are to be indicated first. It is assumed that only one technical object may fail or be repaired at one time. An additional introduced assumption is that all of the objects analysed within the system are homogeneous. Moreover, the application of Markov processes for the estimation of system's availability (and not reliability) requires the use of the system's transition graph without the absorbing state.

Therefore, for the koon systems, when the total number of system's elements is *n*, and the minimal number of the system's elements inevitable for its proper functioning is k, Formula 4 describes the number of the reliability states of the system:

$$m_O = n + 1 \tag{4}$$

where m_S represents the number of the reliability states of the entire system.

Consequently, the number of the operation states of the system may be calculated using Formula 5:

$$m_O = n - k + 1 \tag{5}$$

where m_O represents the number of the operation states of the entire system.

Indication of all of the reliability states of the system in accordance with the possible states of its elements is crucial for the proper development of the system's transition graph. The graph allows one to write down the differential equations related to the probabilities of the occurrence of the indicated states of the system. The general form of the koon system's transition graph, obtained when the above-mentioned assumptions applied, is presented in Fig. 3.

The operation states of the system are, for each case k out of n, the successive states S_1 to S_{n-k+1}. The non-stationary availability coefficient *A(t)* is the probability of the system being in the operational states, and it is estimated after solving the system of Kolmogorov equations. When the course of changes in time of the value of the coefficient *A(t)* becomes constant, then it is the value of the stationary availability coefficient for the analysed koon system.

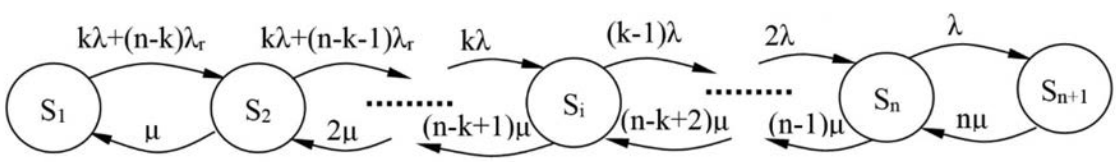

Fig. 3. General form of the koon system's states transition graph without absorbing state

For the case of the structure 4 out of 5, the Kolmogorov equation system, on the basis of Figure 3, takes the following form:

$$\begin{cases} \dfrac{dp_{S1}(t)}{dt} = -(4\lambda+\lambda_r)\,p_{S1}(t) + \mu p_{S2}(t) \\ \dfrac{dp_{S2}(t)}{dt} = -(4\lambda+\mu)\,p_{S2}(t) + (4\lambda+\lambda_r)\,p_{S1}(t) + 2\mu p_{S3}(t) \\ \dfrac{dp_{S3}(t)}{dt} = -(3\lambda+2\mu)\,p_{S3}(t) + 4\lambda p_{S2}(t) + 3\mu p_{S4}(t) \\ \dfrac{dp_{S4}(t)}{dt} = -(2l+3\mu)\,p_{S4}(t) + 3l\,p_{S3}(t) + 4\mu p_{S5}(t) \\ \dfrac{dp_{S5}(t)}{dt} = -(\lambda+4\mu)\,p_{S5}(t) + 2\lambda p_{S4}(t) + 5\mu p_{S6}(t) \\ \dfrac{dp_{S6}(t)}{dt} = -5\mu p_{S6}(t) + \lambda p_{S5}(t) \end{cases} \tag{6}$$

and the non-stationary availability coefficient *A(t)* is calculated using Formula 7:

$$A(t) = p_{S1}(t) + p_{S2}(t) \tag{7}$$

Obviously, the typical use of Markov processes allows one to estimate the availability of a system when its operation times to failure as well as renewal times are described using exponential distribution only. The mentioned feature of the characterised evaluation model is its important disadvantage.

Selected results obtained using the developed simulation model were compared with the results of the analysis conducted for the same systems using Markov processes.

3. Verification of the developed model and the results of exemplary simulation experiments

The results of the availability estimation of 2 out of 3 and 4 out of 5 systems obtained using Markov processes and Matlab software as well as the developed simulation model for the sample data: α = 0,01 [1/t.u.]; λ_r = 0,0025 [1/t.u.]; μ = 0,2 [1/t.u.]; are given in Fig. 4. The probability density function of exponential distribution is presented using Formula 8:

$$f(t) = \lambda exp(-\lambda t) \tag{8}$$

$$\mathrm{f(t)} = \upsilon\beta^{-\nu}t^{\nu-1}\exp\left[-\left(\frac{t}{\beta}\right)^{\upsilon}\right]$$

where $\lambda\upsilon$ – scale parameter.

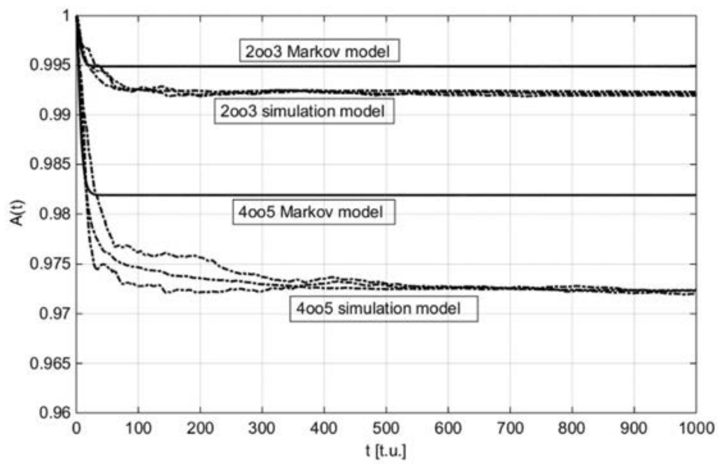

Fig. 4. Availability of 2oo3 and 4oo5 systems obtained using the Markov model and the simulation model

The obtained sample values of the non-stationary availability coefficient (*A(t)*) stabilise in time determining its stationary values (*A*). The differences between the results of the analytical and the simulation solutions are negligible (they differ for the 4oo5 system by around 1% and for the 2oo3 system by around 0.5%), and neither in the presented example nor during the other comparisons performed did they exceeded 2%.

It can be inferred that the developed simulation model works correctly; therefore, it could be applied to estimate the coefficient of the availability of koon systems in which the elements are characterised by non-exponential distributions of the probability of operation and renewal times. An exemplary application of the developed simulation model for such cases was demonstrated using the Weibull distribution. The probability density function of the Weibull distribution is shown using Formula 9:

$$f(t) = \nu\lambda^{\nu}t^{\nu-1}exp\left[-(\lambda t)^{\nu}\right] \tag{9}$$

$$\mathrm{f(t)} = \upsilon\beta^{-\nu}t^{\nu-1}\exp\left[-\left(\frac{t}{\beta}\right)^{\upsilon}\right]$$

where
$\nu\upsilon$ – shape parameter, λ – scale parameter.

The parameters of Weibull distributions used and the reliability structures of the analysed systems are presented in Table 1. The obtained results are shown in Fig. 5.

The results show that improvements in the system's availability may be achieved without altering the structure of the system by improving the renewal intensity (shortening of duration) of damaged components (curves 1, 2, 4 and 5). Changing the reliability structure of the system by adding another redundant element also (to a significant extent) increases the availability of the system (curves 3 and 5). However, the effect obtained from adding another redundant element will be the smaller the greater is the number *n* of elements of the system. In practical applications, this way of improving availability will involve the need to maintain another redundant object. When considering a particular technical system, it is important to make calculations and to take into account the limitations and the technical and economic possibilities to choose the most effective way to provide the required level of availability.

Table 1. Weibull distribution parameters adopted for calculations in simulation model

Curve no. in Fig. 5	System structure	State of element					
		Operation		Reserve		Renewal	
		ν	λ [1/t. u.]	ν	λ [1/t. u.]	ν	λ [1/t. u.]
1	2oo3	3	0.01	3	0.0025	2	0.05
2		3	0.01	3	0.0025	1	0.2
3	4oo5	3	0.01	3	0.0025	1	0.2
4	4oo6	3	0.01	3	0.0025	2	0.05
5		3	0.01	3	0.0025	1	0.2

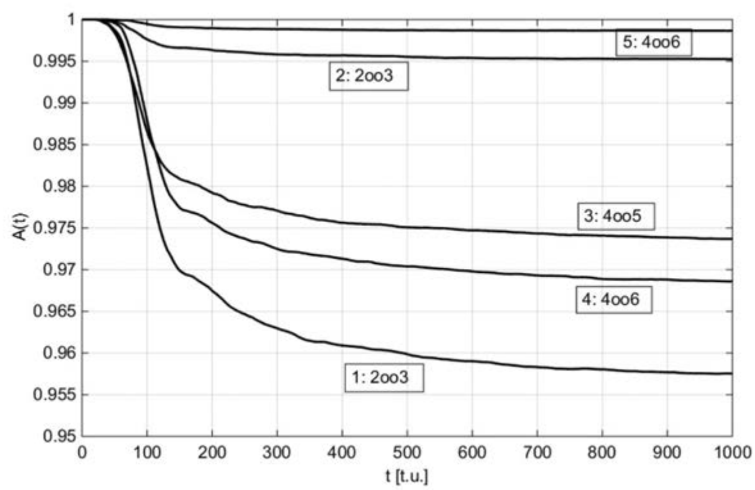

Fig. 5. Availability of 2oo3, 4oo5, and 4oo6 systems obtained in simulation calculations

Conclusions

The presented simulation model for the estimation of complex koon technical systems availability coefficient has been positively verified in the cases where it was possible to apply the widely-known methods based on the Markov processes in parallel. The main advantage of the developed model lies in the huge possibilities of its application when the operation and/or renewal times of the objects incorporated in the analysed system are characterised by non-exponential probability distributions. Such a situation occurs in many practical cases, even when damaged objects are repaired and not replaced with new ones. The useful feature of the developed method is the ability to estimate the availability of real systems and to verify whether they provide its required level. Such requirements are applied, among others, to the systems of pumps and valves used in industrial installations, equipment, and vehicles in fire brigades, emergency services, signalling systems in railways, as well as many systems in the power industry.

Based on the results obtained, it is possible to improve the availability of the system by changing the type of structure koon, i.e. by generally increasing the number of redundant objects in the real system. The second way is to take action to reduce the time of renewal of damaged objects. Improved system's availability can also be achieved by introducing objects characterised by better reliability indexes. An important advantage of the developed computational model is the ability to analyse and search for the most efficient method of increasing the availability of the system to its required level.

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References

1. Gnedenko B., Ushakov I.: Probabilistic reliability engineering. John Wiley & Sons, Inc. New York,

Chichester, Brisbane, Toronto, Singapore, 1995.
2. Morant A., Gustafson A., Soderholm P., et al.: Safety and availability evaluation of railway operation based on the state of signalling systems. Proceedings of the Institution of Mechanical Engineers Part F-Journal of Rail and Rapid Transit, 2017; 231 (2): 226–238.
3. Pariaman H., Garniwa I., Surjandari I., et al.: Availability analysis of the integrated maintenance technique based on reliability, risk, and condition in power plants. International Journal of Technology, 2017; 8 (3): 497–507.
4. Pilch R.: A method for obtaining the required system reliability level by applying preventive maintenance. Simulation: Transactions of the Society for Modeling and Simulation International, 2015; 91 (7): 615–624.
5. Rymarz J., Niewczas A., Krzyżak A.: Comparison of operational availability of public city buses by analysis of variance. Eksploatacja i Niezawodnosc – Maintenance and Reliability, 2016; 18 (3): 373–378.
6. Shaikh A., Mettas A.: Application of reliability, availability, and maintainability simulation to process industries: a case study, [in]: Simulation methods for reliability and availability of complex systems, [edited by]: Faulin J., Juan A.A., Martorell S., et al. Springer Series in Reliability Engineering, 2010: 173–197.
7. Shim J., Ryu H., Lee Y.: Availability analysis of series redundancy models with imperfect switchover and interrupted repairs. Eksploatacja i Niezawodnosc – Maintenance and Reliability, 2017; 19 (4): 640–649.
8. Tang S., Guo X., Sun X., Xue H. and Zhou Z.: Unavailability analysis for *k*-out-of-*n*: G systems with multiple failure modes based on micro-Markov models. Mathematical Problems in Engineering, 2014; article ID740936, 12 pages, http://dx.doi.org/10.1155/2014/740936.
9. Yi Y., Chao R. S., Fan G., et al.: Numerical simulation on the existence of fluctuation of instantaneous availability. Transactions of the Canadian Society for Mechanical Engineering, 2016; 40 (5): 703–713.
10. Zhang T., Long W., Sato Y.: Availability of systems with self-diagnostic components – applying Markov model to IEC 61508-6. Reliability Engineering and System Safety, 2003; 80: 133–141.